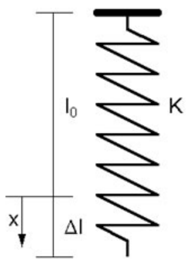
Final exam. Part 1.

1. An extension spring is extended Δl more than its free length l_0 by an external force. If the spring constant is *k*, which of the following expressions provides the work needed to bring the spring to its present state:



	A) $W = 2\,k\,\Delta l$
	B) $W = \frac{1}{2}\,k\,\Delta l$
	C) $W = 2\,k\,(\Delta l)^2$
X	**D)** $W = \frac{1}{2}\,k\,(\Delta l)^2$

2. What of the following statements is the definition of the calorie?

	A) The amount of work require to raise the temperature of 1 gram of water 1 ºC when the water is at 0º C.
	B) The amount of heat require to raise the temperature of 1 kilo-gram of water 1 ºC when the water is at 4º C.
X	**C)** The amount of heat require to raise the temperature of 1 gram of water 1 ºC when the water is at 4º C.
	D) The amount of heat require to raise the temperature of 1 kilo-gram of water 1 ºC when the water is at 0º C

3. If a thermodynamic system is composed of a pure substance, an is able to interchange work in a reversible way with its environment in only one mode, what is the number of independent variables or system properties required to fully knowing the state of that system?

	A) One
X	**B)** Two
	C) Three
	D) None of the above

4. Which of the laws of thermodynamics establishes entropy as a non-conservative magnitude?

	A) Zeroth Law
	B) First Law
X	**C)** Second Law
	D) None of the above

5. Which of the laws of thermodynamics establishes exergy as conservative magnitude?

	A) Zeroth Law
	B) First Law
	C) Second Law
X	**D)** None

6. A gas performs 4 kJ of work after 4 kJ of heat is transferred to the gas from its surroundings. What is the change in internal energy of the gas as a result of this process? Is the process possible?

	A) The change in internal energy is 0.6 J and the process is possible -it does not violate the Laws of Thermodynamics.
	B) The change in internal energy is 600 J and the process is impossible –it violates the First Law of Thermodynamics.
	C) There is no change in internal energy and the process is impossible –it violates the Second Law of Thermodynamics by completely converting heat into work
X	**D)** The change in internal energy is 0 J and the process is possible –it does not violate the Laws of Thermodynamics.

7. Which of the following is an essential characteristic of a pure substance?

	A) It is a simple chemical element
X	**B)** Its uniform in chemical composition and stable
	C) It is only present in a given aggregation state at a given instant in time
	D) It may react chemically with its environment

8. A perfect gas is known to have a gas constant, R, and a ratio of specific heats, k. In terms of these two constants, which of the following set expressions of cp and cv is true?

	A) $c_p = \frac{R}{k-1}$
X	**B)** $c_p = \frac{k}{k-1}R$
	C) $c_p = \frac{2k}{k-1}R$
	D) $c_p = \frac{c_v}{k-1}$

9. Which of the following statements reflects the difference between and ideal gas and a perfect gas?

	A) A perfect gas is similar to an ideal gas. There is no real distinction between both concepts.
	B) The thermal equation of state of a perfect gas and that of an ideal gas are quite different.
	C) A perfect gas is an actual, real gas, which follows the thermal equation of state for ideal gases.
X	**D)** The only difference between an ideal and a perfect gas is that the latter has constant specific heats at constant pressure and volume.

10. Define Exergy

Exergy or Available Energy is a measure of the work potential of a system in a given state and a given environment. It measures the maximum possible work that can be obtained from that system until it reaches the equilibrium with its environment.

11: Starting from one of the forms of the fundamental equation of Thermodynamics for a simple substance, derive the expression for the change in entropy between two equilibrium states of a perfect gas as a function of the specific heat at constant volume, the temperatures and the volumes.

In terms of the variation of internal energy, the Fundamental Equation of Thermodynamics for a simple compressible substance is:

$$du = T\ ds - p\ dv \tag{1}$$

For a perfect gas the following relevant relations hold:

$$du = c_v\ dT \tag{2}$$

$$p\ v = R\ T \Rightarrow \frac{p}{T} = \frac{R}{v} \tag{3}$$

Combining (1) and (2):

$$ds = c_v\ \frac{dT}{T} + \frac{p}{T}\ dv \tag{4}$$

Combining (3) and (4):

$$ds = c_v\ \frac{dT}{T} + R\ \frac{dv}{v} \tag{5}$$

Integrating (5) between the initial and final equilibrium state:

$$\int_1^2 ds = c_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{v_1}^{v_2} \frac{dv}{v} \Rightarrow s_2 - s_1 = c_v\ \ln\frac{T_2}{T_1} + R\ \ln\frac{v_2}{v_1} \tag{6}$$

2. Starting from one of the forms of the fundamental equation of Thermodynamics for a simple substance, derive the expression for the change in entropy between two equilibrium states of a perfect gas as a function of the specific heat at constant pressure, the temperatures and the pressures.

In terms of the variation of enthalpy, the Fundamental Equation of Thermodynamics for a simple compressible substance is:

$$dh = T\ ds + v\ dp \tag{1}$$

For a perfect gas the following relevant relations hold:

$$dh = c_p\ dT \tag{2}$$

$$p\ v = R\ T \Rightarrow \frac{v}{T} = \frac{R}{p} \tag{3}$$

Combining (1) and (2):

$$ds = c_p\ \frac{dT}{T} - \frac{v}{T}\ dp \tag{4}$$

Combining (3) and (4):

$$ds = cp\ \frac{dT}{T} - R\ \frac{dp}{p} \tag{5}$$

Integrating (5) between the initial and final equilibrium state:

$$\int_1^2 ds = c_p \int_{T_1}^{T_2} \frac{dT}{T} - R \int_{v_1}^{v_2} \frac{dp}{p} \Rightarrow s_2 - s_1 = c_v\ \ln \frac{T_2}{T_1} - R\ \ln \frac{p_2}{p_1} \tag{6}$$

Final exam. Part 2.

Instructions: Select one problem from Problem Group A and another from Problem Group B. You have to solve two problems in total, one from each group of problems.

<u>Problem Group A</u>

Problem A1. Consider a Carnot heat-engine cycle executed in a closed system using 0.0103 kg of steam as the working fluid. It is known that the maximum absolute temperature in the cycle is twice the minimum absolute temperature, and the work output of the cycle is 25 kJ. If the steam changes form saturated vapor to saturated liquid during heat rejection, determine the temperature of the steam during heat rejection.

The efficiency of the Carnot heat-engine is:

$$h = 1 - \frac{T_L}{T_H} \tag{1}$$

where T_H and T_L are the maximum and the minimum absolute temperatures in the cycle, respectively.

Since we are told that the maximum absolute temperature is twice the minimum,

$$h = 1 - \frac{T_L}{2\,T_L} = 1 - \frac{1}{2} = \frac{1}{2} \tag{2}$$

From the definition of efficiency,

$$h = \frac{W}{Q_H} \;\Rightarrow\; Q_H = \frac{W}{h} = \frac{W}{1/2} = 2\,W \tag{3}$$

Applying the First Law to the operation of the Carnot heat-engine in one cycle,

$$W = Q_H - Q_L = 2\,W - Q_L \;\Rightarrow\; Q_L = W \tag{4}$$

Thus, the heat rejected by the Carnot engine per cycle in the process of going from saturated steam vapor to saturated liquid vapor is equal to the work output of the cycle, i.e. 25 kJ.

Since this process of condensation is not only a process at constant temperature, but also at constant pressure, the heat released during it has to be equal to the enthalpy of phase-change (from liquid to vapor) at the unknown minimum absolute temperature of the Carnot engine cycle.

Thus,

$$Q_L = m\,h_{fg}(T) \tag{5}$$

Combining (4) and (5) and solving for h_{fg},

$$h_{fg}(T) = \frac{W}{m} \;\Rightarrow h_{fg}(T) = \frac{25\,kJ}{0.0103\,kg} = 2427.18\,\frac{kJ}{kg} \tag{6}$$

Using the Tables for Saturated Water the following linear interpolation can be constructed,

$$T = \frac{T_B - T_A}{h_{fg}(T_B) - h_{fg}(T_A)}[h_{fg}(T) - h_{fg}(T_A)] + T_A \tag{7}$$

Replacing values,

$$T = \frac{35-30}{2418.62-2430.48}[2427.18-2430.48]+30 = 31.39\,^{o}C \Rightarrow T = 304.54\,K \quad (8)$$

Thus, the minimum absolute temperature in the cycle is 304.54 K.

Problem A2. Consider two Carnot engines operating in series. The first engine receives heat from the reservoir at 2400 K and rejects the waste heat to another reservoir at temperature T. The second engine receives this energy rejected by the first one, and converts some of it to work, and rejects the rest to a reservoir at 300 K. If the thermal efficiencies of both engines are the same, determine the temperature T.

The thermal efficiency of the first Carnot engine can be written as

$$h_1 = 1 - \frac{T}{T_H} \quad (1)$$

where T_H is 2400 K.

Likewise, the thermal efficiency of the second Carnot engine can be written as

$$h_2 = 1 - \frac{T_L}{T} \quad (2)$$

where T_L is 300 K.

Imposing the condition that both efficiencies are the same, the following expression is obtained.

$$h_1 = h_2 \quad \Rightarrow 1 - \frac{T}{T_H} = 1 - \frac{T_L}{T} \quad \Rightarrow \frac{T}{T_H} = \frac{T_L}{T} \quad (3)$$

Solving for T,

$$T = \sqrt{T_H \times T_L} = \sqrt{2400 \times 300} = 848.53\,K \quad (4)$$

<u>Problem Group B</u>

Problem B1. Air is compressed by a 12 kW compressor from P1 to P2. The air temperature is mantained constant at 25 ºC during this process as a result of heat transfer to the surrounding medium at 10 ºC. Determine the rate of entropy change of the air. Assume that air is a perfect gas, and that there is no entropy generation within the compressor.

The assumption of perfect gas behaviour implies, among other things, that the energy and the enthalpy of the gas are function of temperature exclusively. Thus, since the air temperature is kept constant, the enthalpy of the air at the entrace of the compressor must be equal to its enthalpy at the exit. Therefore,

$$h_i = h_e = h(25\,^{o}C) \quad (1)$$

Under the assumption of steady state conditions, the First Law of Thermodynamics yields

$$\dot{m}\,h_i + \dot{W} = \dot{m}\,h_e + \dot{Q}_{Loss} \quad (2)$$

Combining (1) and (2),

$$\dot{W} = \dot{Q}_{Loss} \tag{3}$$

Under the assumption of steady state conditions, the entropy balance for the compressor yields,

$$\dot{S} = \frac{-\dot{Q}_{Loss}}{T_{compressor}} \tag{4}$$

Combining (3) and (4) and entering numerical values,

$$\dot{S} = \frac{-\dot{W}}{T_{compressor}} = -\frac{12\ kW}{(25 + 273.15)\ K} = -\frac{12\ kW}{298.15\ K} = -\,0.0402\,\frac{kW}{K} \tag{4}$$

Thus, the rate of entropy change of the air is –0.0402 kW/K.

ProblemB2. A rigid tank is divided into two equal parts by a partition. One part of the tank contains 1.5 kg of compressed liquid water at 300 kPa and 60 ºC while the other part is evacuated. The partition is now removed, and the water expands to fill the entire tank. Determine the entropy change of the water during this process, if the final pressure in the tank is 15 kPa.

In the initial state, the properties of the water are:

$$\left.\begin{array}{l} p_1 = 300\ kPa \\ T_1 = 60\ ^{\circ}C \end{array}\right\} \Rightarrow \begin{array}{l} v_1 \cong v_{f,sat}(60\ ^{\circ}C) = 0.001017\ \dfrac{m^3}{kg} \\ s_1 \cong s_{f,sat}(60\ ^{\circ}C) = 0.8311\,\dfrac{kJ}{kg\ K} \end{array} \tag{1}$$

Since in the final state the total volume occupied by the water is twice the initial volume and the amount of water remains the same,

$$v_2 = 2\ v_1 = 2 \times 0.001017 = 0.002034\ \frac{m^3}{kg} \tag{2}$$

Thus, in the final state, the properties of the water are:

$$\left.\begin{array}{l} p_2 = 15\ kPa \\ v_2 = 0.002034\dfrac{m^3}{kg} \end{array}\right\} \begin{array}{l} x = \dfrac{v_2 - v_{f,sat}(15\ kPa)}{v_{fg}(15\ kPa)} = \dfrac{0.002034 - 0.001014}{10.02117} = 0.0001018 \\ s_2 = s_{f,sat}(15\ kPa) + x\ s_{fg}(15\ kPa) = 0.7548 + 0.0001018 \times 7.2536 = 0.7556\ \dfrac{kJ}{kg\ K} \end{array}$$

Then, the entropy change of the water is,

$$\Delta S = m\,(s_2 - s_1) = 1.5\ kg\ (0.7556 - 0.8311)\ \frac{kJ}{kg\ K} = -\,0.1134\ \frac{kJ}{K} \tag{4}$$